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**FITPI: Modifications to the Optical Model  
Program PIRK for Parameter Searches and  
Total Cross Sections**

by

M. D. Cooper  
R. A. Eisenstein\*



\*Visiting staff member. Permanent address: Carnegie-Mellon University,  
Pittsburgh, PA 15213. Mr. Eisenstein's work supported by ERDA  
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FITPI

MODIFICATIONS TO THE OPTICAL MODEL PROGRAM

PIRK

FOR PARAMETER SEARCHES AND TOTAL CROSS SECTIONS

by

M. D. Cooper and R. A. Eisenstein



ABSTRACT

The pion-nucleus optical code PIRK has been modified to allow for searches on the optical potential parameters and nuclear density and charge parameters. The code can simultaneously fit both to positive and negative pion elastic scattering angular distributions as well as total cross sections, total reaction cross sections, and total elastic cross sections. The operation of the code has been changed to separate more fully Coulomb from nuclear effects, hence allowing for the calculation of total cross sections.

I. INTRODUCTION

The program has been fully described in the PIRK users' manual,<sup>1</sup> but for convenience the basic outline of the code is included here. Though this work is intended to be self-contained, those wishing a more complete understanding of the programming details are referred to Ref. 1.

The program PIRK is a computer code which solves the potential scattering problem of a relativistic spin zero particle interacting with a complex optical potential. It does this by numerically integrating a Klein-Gordon equation for each partial wave (which includes both the Coulomb and complex optical potential) from the origin outwards to a predetermined match point, where the "internal" logarithmic derivatives are compared to those from the "external" or asymptotic Coulomb wave functions for that partial wave. Phase shifts are thus obtained for each partial wave and are used to calculate the differential elastic cross section and the total reaction cross section.

II. THEORETICAL DESCRIPTION

The Klein-Gordon equation for stationary eigenstates is obtained from the relativistic energy relation

$$E^2 - p^2 c^2 = m_0^2 c^4 \quad (1)$$

by the usual replacement  $\vec{p} = -i\hbar\vec{\nabla}$  for the relativistic 3-momentum. We may include<sup>2</sup> the electromagnetic potentials  $\vec{A}(\vec{r})$  and  $\phi(\vec{r})$  in the wave equation by using the fact that  $\phi$  and  $\frac{1}{c}\vec{A}$  have the same Lorentz-transformation properties as  $E$  and  $\vec{p}$ , namely as the time and space parts of a 4-vector, respectively.

Equation (1) reads then

$$(E - e\phi)^2 - (\vec{p} - \frac{e}{c}\vec{A})^2 c^2 = m_0^2 c^4. \quad (2)$$

Here  $E$  is the total energy,  $m_0$  the particle rest mass,  $\vec{p}$  the particle 3-momentum. In the treatment that follows, the electromagnetic interaction between the charged pions and a nucleus is purely electrostatic and may be represented by a scalar potential  $\phi(\vec{r})$  and vector potential  $\vec{A}(\vec{r}) = 0$ . The potential  $\phi(\vec{r})$  will be computed from a finite-sized spherically symmetric nuclear charge distribution. In what follows,  $V_c = e\phi$ .

The structure of Eq. (1) shows that a potential energy term cannot be arbitrarily added to Eq. (2) to account for, say, the strong interaction between pions and nuclei. The Lorentz transformation properties of such terms must be investigated first. Lorentz invariant parts would be included with  $m_0 c^2$ , parts transforming like 4th components of 4-vectors would be included with  $E$ , and so on. Because the pion-nucleus optical potential is not completely understood, it has been commonly assumed that it can be included with the energy  $E$  and the Coulomb potential:<sup>3</sup>

$$E \rightarrow E - V_C - V_N$$

$$E^2 \approx (E - V_C)^2 - 2EV_N.$$

In addition, the terms  $2V_C V_N$  and  $V_N^2$  are arbitrarily dropped from the expression for  $E^2$ .

The Klein-Gordon equation now is

$$\begin{aligned} & \{ -(\hbar c \nabla_r)^2 + \frac{c^2 \hat{L}^2}{r^2} + (m_0 c^2)^2 \} \psi \\ &= \{ E^2 - 2V_C E + V_C^2 - 2EV_N \} \psi. \end{aligned}$$

The operator  $\hat{L}^2$  is the usual one for the square of the orbital angular momentum, while  $\nabla_r$  is the radial part of the gradient operator. We make the following substitutions and definitions:

$$\psi = \sum_{\ell} \frac{u_{\ell}(r)}{r} i^{\ell} P_{\ell}(\cos \theta) ; \quad U_C = \frac{V_C}{\hbar c} ; \quad U_N = \frac{V_N}{\hbar c} ;$$

$$\xi = \frac{E}{\hbar c} ;$$

$$k^2 = (p/\hbar)^2 = \frac{E^2 - m_0^2 c^4}{(\hbar c)^2}$$

and find that the Klein-Gordon equation is

$$\begin{aligned} & \left[ -\nabla_r^2 + \frac{\ell(\ell + 1)}{r^2} \right] \frac{u_{\ell}}{r} Y_{\ell m} \\ &= [k^2 + U_C^2 - 2\xi(U_C + U_N)] \frac{u_{\ell}}{r} Y_{\ell m}. \end{aligned}$$

Several forms have been introduced in the literature for the optical potential  $U_N$ . These include the Kisslinger form,<sup>4</sup> the Laplacian<sup>5</sup> (local) form, and an "improved" Kisslinger form.<sup>6</sup> In order to have these various models available in the code, we define

$$2\xi U_N \psi = [A_1 \rho + A_2 \vec{\nabla} \cdot \rho \vec{\nabla} + A_3 \nabla^2 \rho + A_4 \nabla^4 \rho] \psi. \quad (3)$$

Proper choices of constants  $A_1, A_2, A_3, A_4$  encompass all models mentioned above. The quantity  $\rho$  is the nuclear matter density which we assume to be spherically symmetric:  $\rho = \rho(r)$ . Replacing  $\psi$  as mentioned above and carrying out some algebra we obtain the following radial equation for each partial wave:

$$\begin{aligned} & (A_2 \rho - 1) u_{\ell}'' + A_2 \rho' u_{\ell}' \\ &= \left\{ \frac{A_2 \rho'}{r} + k^2 - 2U_C \xi + U_C^2 - A_3 \nabla_r^2 \rho \right. \\ & \quad \left. - A_4 \nabla_r^4 \rho - A_1 \rho + \frac{\ell(\ell + 1)}{r^2} (A_2 \rho - 1) \right\} u_{\ell}. \quad (4) \end{aligned}$$

In the program Eq. (4) is represented as

$$u_{\ell}'' + f(r) u_{\ell}' + [g(r) - \frac{\ell(\ell + 1)}{r^2}] u_{\ell} = 0 \quad (5)$$

with

$$f(r) = \frac{A_2 \rho'}{A_2 \rho - 1}$$

and

$$\begin{aligned} g(r) = (1 - A_2 \rho)^{-1} & \left\{ \frac{A_2 \rho'}{r} + k^2 - 2U_C \xi + U_C^2 \right. \\ & \quad \left. - A_3 \nabla_r^2 \rho - A_1 \rho - A_4 \nabla_r^4 \rho \right\}. \end{aligned}$$

Notice that  $f(r)$  and  $g(r)$  are independent of  $\ell$  and are complex functions.

Equation (5) is integrated outward numerically, beginning at the origin and stopping at the matching radius. At that point, this "inner" wave function  $u_{\ell}$  is compared to our "outer" wave function that is a solution to the (Coulomb) scattering problem in the absence of the nuclear potential  $U_N$ . The phase shift  $\delta_{\ell}$  is obtained from the relation

$$S_{\ell} = e^{2i\delta_{\ell}} = \left[ \frac{D(F + iG) - (F' + iG')}{(F' - iG') - D(F - iG)} \right]_{\ell}. \quad (6)$$

The primes indicate differentiation with respect to  $r$ . The quantities  $F$  and  $G$  are the regular and irregular Coulomb wave functions,<sup>7</sup> respectively,  $S_{\ell}$  is the S-matrix element, and  $D$  is the logarithmic derivative of the inner wave function at the match point:

$$D = \frac{u_{\ell}'}{u_{\ell}} \Big|_{r = \text{match point}}$$

Note that we are using Coulomb wave functions that are solutions to the non-relativistic Schrödinger equation instead of the "relativistic" Coulomb

functions which arise when solving Eq. (4) with no nuclear potential. The error is largest for s states but is not large even then. The stability of the cross sections as a function of matching radius has been checked and confirms this. The slight error is more than offset by the relative ease of computing non-relativistic Coulomb wave functions over their relativistic counterparts.

The phase shifts  $\delta_\ell$  are complex because of the complex nuclear potential and therefore  $S_\ell$  is less than the unitary limit. The differential and reaction cross sections are calculated from the following formulas:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$f(\theta) = f_C(\theta) + f_N(\theta)$$

$$f_C(\theta) = -\frac{\eta}{2p \sin^2(\theta/2)} \exp\{2i[\sigma_0 - \eta \ln(\sin \frac{\theta}{2})]\}$$

$$\eta = Z_N Z_\pi \alpha(\frac{E}{p})$$

$$f_N(\theta) = \frac{i}{2k} \sum_{\ell=0}^{LMAX-1} (2\ell + 1) S_{C\ell}(1 - S_{N\ell}) P_\ell(\cos \theta)$$

$$S_{C\ell} = e^{2i\sigma_\ell} \quad S_{N\ell} = e^{2i\delta_{N\ell}} \quad (7)$$

$\theta$ ,  $E$ ,  $p$ , and  $k$  are the center of mass scattering angle, total energy, pion momentum and wave number  
 $\delta_\ell$  = nuclear phase shift for  $\ell^{\text{th}}$  partial wave in presence of Coulomb potential  
 $\sigma_\ell$  = point charge Coulomb phase shift of  $\ell^{\text{th}}$  partial wave  
 $\alpha = e^2/hc$

### III. MODIFICATIONS TO PIRK

#### A. Definition of Total Cross Section and Total Elastic Cross Section

The presence of the Coulomb force makes the integrated elastic scattering cross section divergent, and so it is necessary to define a total cross section which is finite and consistent with the one commonly adopted by experimenters. If we separate the full amplitude as done in Eq. (7) into a Coulomb term and a residual piece, which we hope is dominated by nuclear processes, we can define the total nuclear elastic scattering cross section as

$$\sigma_{\text{el}} = \frac{4\pi}{\Omega} [ |f(\theta)|^2 - |f_C(\theta)|^2 - 2\text{Re}f_C^*(\theta)f_N(\theta) ]$$

$$= \frac{4\pi}{\Omega} |f_N(\theta)|^2. \quad (8)$$

The total cross section is then given by

$$\sigma_{\text{Tot}} = \sigma_{\text{el}} + \sigma_{\text{react}} \quad (9)$$

where we have the following helpful formulas to aid in the calculation

$$\sigma_{\text{react}} = \frac{\pi}{k^2} \sum (2\ell+1) (1 - |S_{N\ell}|^2) \quad (10a)$$

$$\sigma_{\text{Tot}} = \frac{2\pi}{k^2} \sum (2\ell+1) \text{Re}(1 - S_{N\ell}). \quad (10b)$$

#### B. Modifications for the Separation of Coulomb and Nuclear Effects

Since PIRK solves a Klein-Gordon equation for which normal Coulomb wave functions are not solutions, there is a modification to the elastic scattering due to the  $V_c^2$  term in the potential. Additional changes are caused by finite charge distributions. In order to calculate a total cross section and interference term  $[2\text{Re}f_C^*(\theta)f_N(\theta)]$  which do not erroneously assume a pointlike charge distribution, the following modifications have been made. The full amplitude in Eq. (7) is now written

$$f = f_{C0} + f_{C1} + f_N \quad (11)$$

where

$$f_{C0} = \frac{i}{2k} \sum_{\ell} (2\ell+1) P_\ell(1 - S_{C0\ell}) \quad (12a)$$

is due to a pointlike charge distribution,

$$f_{C1} = \frac{i}{2k} \sum_{\ell} (2\ell+1) P_\ell S_{C0\ell}(1 - S_{C1\ell}) \quad (12b)$$

is the correction for relativity and finite charge, and

TABLE I  
CHARGE DISTRIBUTIONS AND SCALAR POTENTIALS

	<u>Charge Distribution</u>	<u>Potential</u>
Uniform:	$\rho(r') = \begin{cases} \frac{3Ze}{4\pi R_c^3} & r' < R_c \\ 0 & r' > R_c \end{cases}$	$\phi(R) = \begin{cases} \frac{Ze^2}{2R_c} (3-R^2/R_c^2) & R < R_c \\ Z/R & R > R_c \end{cases}$
Modified Gaussian:	$\rho(r') = \frac{2e}{(\sqrt{\pi} R_c)^3} \exp[-(r'/R_c)^2] (1 + \frac{Z-2}{3} \frac{r'}{R_c})^2$	$\phi(R) = \frac{Ze^2}{R} \operatorname{erf}(R/R_c) - \frac{(2Z-4)e^2}{3\sqrt{\pi} R_c} \exp[-(R/R_c)^2]$
Woods-Saxon:	$\rho(r') = \frac{3Ze}{4\pi R_c^3 [1 + (\frac{r'}{R_c})^2]} \cdot \frac{1}{1 + \exp[(r' - R_c)/a_c]}$	$\phi(R) = \frac{Ze^2}{R} + \frac{3Za_c^2 e^2}{R_c^3 [1 + (\frac{R_c}{R})^2]} \int_{R/a}^{\infty} dx \frac{x(1 - \frac{x a_c}{R})}{1 + \exp(x - R_c/a_c)}$

$$f_N = \frac{i}{2k} \sum_{\ell} (2\ell+1) P_{\ell} S_{C0\ell} S_{C1\ell} (1 - S_{N\ell}) \quad (12c)$$

is due to the residual interaction in presence of Coulomb distortion.

To get the  $S_{C1\ell}$ , the computer program calculates the wave function for the finite potential by integrating the relativistic Klein-Gordon equation and matching to non-relativistic Coulomb functions. Hence, both relativistic effects and finite charge effects are included in the  $S_{C1\ell}$ . The nuclear potential is taken to be zero.

To get the  $S_{N\ell}$ , the program calculates the relativistic Klein-Gordon equation and matches to

$$F_{\ell} \cos \sigma_{C1\ell} + G_{\ell} \sin \sigma_{C1\ell} \quad \text{regular solution}$$

$$- F_{\ell} \sin \sigma_{C1\ell} + G_{\ell} \cos \sigma_{C1\ell} \quad \text{irregular solution.}$$

#### C. Spherical Charge Distributions and Scalar Potentials

Gauss' Law for a spherical charge distribution gives the radial electric field as

$$E(r) = \frac{4\pi}{r^2} \int_0^r \rho(r') r'^2 dr' \quad (13a)$$

and the scalar potential can be derived from Eq. (13a) by

$$\phi(R) = \int_R^{\infty} E(r) dr \quad (13b)$$

If we require that the charge density  $\rho(r')$  be normalized to  $Z = \int dr \rho(r')$ , then we can calculate some useful potentials from Eq. (13), and they are listed in Table I.

#### IV. RUNNING THE PROGRAM

A guide to the input parameters and program options is given below. A sample run is also presented.

When running the program one must always be sure to include enough partial waves in the amplitude sum. As a guide, Table II is presented. Real ( $S_{\ell}$ ) > .999 was used as a cut off value; the spaces for each element and energy contain the number of partial waves required in calculations for  $\pi^+$ . Failure to include enough partial waves will cause spurious oscillations in  $d\sigma/d\Omega$  at high energies and back angles (and possibly other maladies).

TABLE II

THE TABLE CONTAINS, FOR SEVERAL ENERGIES AND TARGET NUCLEI, THE NUMBER OF PARTIAL WAVES REQUIRED FOR GOOD CONVERGENCE OF THE PARTIAL WAVE SUM (THE NUCLEAR SCATTERING AMPLITUDE)

Energy (MeV)	$^{12}\text{C}$	$^{40}\text{Ca}$	$^{60}\text{Ni}$	$^{120}\text{Sn}$	$^{208}\text{Pb}$
75	6	8	9	10	11
100	7	9	10	12	13
200	11	16	17	18	19
300	13	19	20	22	24

Program output consists of the fixed parameters of the calculation, the final search parameters of the calculation, and the fits to the experimental data. For each partial wave the quantities

$$F\text{-SUB-L} = \frac{1}{2}(e^{2i\delta} l - 1)$$

and

$$S\text{-SUB-L} = e^{2i\delta} l$$

are printed for the final fit. A graphical display of angular distributions and fits is given.

Sample output for the case of 75-MeV  $\pi$  scattering from  $^4\text{He}$  is presented in Appendix A. The test deck shown in Fig. 1 runs this case.

#### A. The Search Routines

The program contains an option regarding the type of non-linear least squares search made. The choices trade off speed for size of convergence region when minimizing

$$\chi^2 = (\text{calculation-data})^2 / (\text{standard deviation})^2.$$

The fastest and most economical technique is an approximate quadratic method written by N. R. Yoder.<sup>8</sup> It expands the  $\chi^2$  space in terms of first and second derivatives with respect to the parameters. The second derivatives are approximated by keeping terms quadratic in the first derivatives. A step is taken based on the predicted direction of the minimum in  $\chi^2$  and a check is made for improvement. If  $\chi^2$  is improved, further steps are made and if no improvement is found, then partial steps are taken until convergence is reached. The total number of steps or cycles is limited in the program input. The method is similar to the technique described on page 222 of Bevington.<sup>9</sup>

When near the minimum, this quadratic extrapolation is quite rapid. If one is far from the solution and a parabolic approximation is not valid, the search can actually go wrong. For example, a very deep canyon in  $\chi^2$  space which is parabolic only at the bottom will be very difficult to approach by this technique. The usual symptoms are seeing the first step go to a larger  $\chi^2$  than the initial calculation and even having the partial steps fail.

To rectify this situation, an alternate search method is provided which uses a linear expansion in the parameters of the calculated data and mixes in an appropriate amount of gradient search to insure that  $\chi^2$  decreases. The technique is that shown on page 232 of Bevington.<sup>9</sup> This method has the advantage of having a larger region of convergence, but at the cost of speed. Its linearization causes the final minimum to be approached quite slowly even though it is in the neighborhood. One may discover that starting the search with this technique and finishing with the parabolic approach is most efficient.

Experience shows that the first technique will probably converge within about 5 steps, whereas considerably more steps may be needed for the second. The user should always check the search history to see that a true minimum has been reached.

To do a one-shot calculation and check the  $\chi^2$  of the calculation for a set of parameters, use the first search technique with the number of cycles set to 0.

#### B. Program Input and Options

Card 1 (16I5) JPAR(1), JPAR(2), ..., JPAR(10)  
(a blank card stops program)

If JPAR(I)=0, this parameter not searched on  
=1, this parameter is searched on

I = 1 Re $b_0$	I = 6 Im $b_2$
2 Im $b_0$	7 A
3 Re $b_1$	8 T
4 Im $b_1$	9 AZ = $R_c / (AN)^{1/3}$
5 Re $b_2$	10 TZ

For definitions, see cards 3 through 5.

Card 2 (16I5) JDATA(1), JDATA(2), ..., JDATA(8)  
 JDATA(I) selects the type of data to be fit. (JDATA(I), I=1,6) are restricted to 0 or 1.

If JDATA(1)=1  $\sigma_{\text{Tot}}(\pi+)$  part of fitted data  
 (2)=1  $\sigma_{\text{Tot}}(\pi-)$  part of fitted data  
 (3)=1  $\sigma_{\text{el}}(\pi+)$  part of fitted data  
 (4)=1  $\sigma_{\text{el}}(\pi-)$  part of fitted data  
 (5)=1  $\sigma_{\text{react}}(\pi+)$  part of fitted data  
 (6)=1  $\sigma_{\text{react}}(\pi-)$  part of fitted data

Above quantities defined in Eqs. (8-10)

(7)=nang Number of angles to fit  
 with  $\pi+$  elastic scattering.  
 (8)=nang Number of angles to fit  
 with  $\pi-$  elastic scattering.

Card 3 (1615) LDX, NCY, LMAX, NPOT, NDEN, NCHAR, NSEAR

LDX = Number of mesh points not including zero but including the match point. (Typical value = 200)  $LDX \leq 500$ . LDX = 0 stops program.

NCY = Number of search steps (Typical is 5)  
 NCY=0 calculates cross sections at data points without search. For NSEAR=2, more steps are usually required.

LMAX= Number of partial waves to be calculated.  
 ( $\ell$  varies from  $\ell = 0$  to  $\ell = LMAX - 1$ )

NPOT= Choice of nuclear optical potential.  
 (Default = 1)

Blank	Default
1	$V_N(r) = -Ak^2b_0\rho(r) + Ab_1^2\vec{V}\cdot\vec{\rho}$ Standard Kisslinger form <sup>3</sup>
2	$V_N(r) = -Ak^2(b_0 + b_1)\rho(r)$ $-A\frac{b_1}{2}\nabla^2\rho(r)$ Local (Laplacian) form <sup>4</sup>
3	$V_N(r) = -Ab_0^2k^2\rho(r) + Ab_1\vec{V}\cdot\vec{\rho}$ $-A\frac{E_c + m\pi}{2}b_1^2\nabla^2\rho$ "Modified Kisslinger" form <sup>5</sup>

In the above expressions, A = nuclear mass in amu; k =  $\pi$  - Nucleus c.m. wave number;  $\rho$  = nuclear matter density; m = nucleon mass;  $E_c$  = kinetic energy of  $\pi$  in cm;  $b_0$ ,  $b_1$ , and  $b_2$  are (complex) input parameters.

NDEN = Choice of nuclear matter density.  
 (Default = 1)

Blank	Default
1	$\rho(r) = \frac{2}{Z(\sqrt{\pi}w)^3} \cdot$ $(1 + \frac{Z-2}{3}(\frac{r}{w})^2) \exp\{-\frac{r^2}{w^2}\}$
2	$\rho(r) = \frac{3}{4\pi c^3} (1 + (\frac{\pi r}{c})^2)^{-1} \cdot$ $(1 + \exp\{\frac{r-c}{t}\})^{-1}$ $t = a/4\ln 3 \quad a = 90-10\%$ "skin thickness"
3	$\rho(r) = \frac{3}{4\pi R^3} r^2 \quad r < R$ $= 0 \quad r > R$

For this last distribution,  $\rho'$  and  $\rho''$  are set equal to zero. This  $\rho(r)$  has been used for tests of the code.

NCHAR= Choice of charge distribution, refer to Table 1. (Default=2)

Blank	Default
1	Uniform
2	Modified Gaussian
3	Woods-Saxon

NSEAR= Choice of type of search performed

(Default=1)

Blank	Default
1	Parabolic expansion of $\chi^2$ space
2	Mixed gradient and linearization of $\chi^2$

Card 4 (8F10.0) DR,E,AN, ZZ, C, A, CZ, AZ

Initial values and constants of the search

DR = Grid size (0.05 fermi is a typical value)

E = Pion lab kinetic energy

AN = Atomic mass number

ZZ = Nuclear charge

CZ = (Charge radius/ $A^{1/3}$ ) =  $r_0 = R_c$

RC = (CZ)  $AN^{1/3} = r_0 A^{1/3} = R_c$

AZ =  $a_c$  used only with Woods-Saxon charge distribution

(If  $R_c \leq$  mesh size DR, a point charge distribution results.)

$R_c$  is related to the rms radius of a square well by

$$r_0 A^{1/3} = \left(\frac{5}{3} \langle r^2 \rangle\right)^{1/2}.$$

C = size parameter for the nuclear matter density

C = w (Gaussian)

C = c (Fermi)

C = R (Square Well)

A = "Thickness" parameter for nuclear matter density.

Not used (Gaussian)

A = a (Fermi)

Not used (Square Well)

Card 5 (8F10.0) Re  $b_0$ , Im  $b_0$ , Re  $b_1$ , Im  $b_1$ , Re  $b_2$ .  
Im  $b_2$

Potential parameters-initial values and constants of the search

Re  $b_0$ , Im  $b_0$  = Real and imaginary part of the parameter  $b_0$ .

Re  $b_1$ , Im  $b_1$  = Real and imaginary part of the parameter  $b_1$ .

Re  $b_2$ , Im  $b_2$  = Real and imaginary part of the parameter  $b_2$ .

Card 6 onward (8F10.0): There is one card for each data point if and only if that datum is to be fit.

8  
Total data cards =  $\sum_{I=1}^8$  JDATA (I), e.g., if there

is a fit required to total cross sections and angular distributions at 15 angles for both  $\pi^+$  and  $\pi^-$ , then there are 32 data cards = 1+1+15+15.

For (JDATA(I), I=1,6) = 1, format is: cross section, standard deviation.

For (JDATA(7).OR.JDATA(8)) ≠ 0 format is: angle, differential cross section, standard deviation.

Units for data are millibarns, center-of-mass degrees, and millibarns/steradian where appropriate.

A typical data deck is shown in Fig. 1. The input deck reads from the bottom to the top of the page. Appendix A shows the corresponding output for this case. In the graphical displays of the fits, the X represents the data and the O represents the fit.

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Blank card terminates job unless next case inserted here.

15  $\pi^-$  angular distribution cards (angle, differential, cross section, error).

15  $\pi^+$  angular distribution cards (angle, differential, cross section, error).

$\pi^+$  and  $\pi^-$  total cross section cards (cross section, error).

First five data cards  
described in text.

Fig. 1. A sample data deck for 75-MeV pions on  ${}^4\text{He}$ . The card order is from bottom to top.

## APPENDIX A

## SAMPLE PROGRAM OUTPUT FOR THE CASE OF FIG. 1

CYCLE 0 STEP 0.00 CHISG= 1.2217E+03  
 CYCLE 1 STEP 1.00 CHISG= 7.3250E+02  
 CYCLE 2 STEP 1.00 CHISG= 7.1697E+02  
 CYCLE 3 STEP 1.00 CHISG= 7.1687E+02  
 CYCLE 3 STEP 0.50 CHISG= 7.1687E+02  
 CYCLE 3 STEP 0.25 CHISG= 7.1686E+02

L	F=SUB=L(+) REAL		S=SLB=L(+) REAL		F=SLE=L(-) REAL		S=SLE=L(-) REAL	
	IMAG	IMAG	IMAG	IMAG	IMAG	IMAG	IMAG	IMAG
0	43340E+02	15234E+00	10287E+01	30468E+00	12916E-02	14725E+00	10024E+01	29459E+00
1	19615E+00	14002E+00	6762E+00	29623E+00	22730E+00	14885E+00	58539E+00	29779E+00
2	20657E+01	42194E+01	95869E+00	84388E+01	22865E+01	45243E+01	95427E+00	98085E+01
3	79783E+03	36252E+02	95942E+00	72524E+02	88158E+03	39482E+02	99824E+00	78563E+02
4	29722E+04	21914E+03	99994E+00	42029E+03	32702E+04	22997E+03	99993E+00	45594E+03
5	11493E+05	97700E+05	100000E+01	19400E+04	12637E+05	12642E+04	100000E+01	21296E+04
6	41626E+07	37284E+06	100000E+01	74560E+06	46244E+07	41225E+06	100000E+01	22851E+06
7	13594E+08	12225E+07	100000E+01	24450E+07	14890E+08	13478E+07	100000E+01	26956E+07
8	38331E+10	34843E+09	100000E+01	69687E+09	42327E+10	38475E+09	100000E+01	76546E+09
9	96392E+12	87698E+11	100000E+01	17542E+12	12650E+11	96959E+11	100000E+01	19392E+12

## OPTICAL MODEL PARAMETER SEARCH TO FIT DATA

## CALCULATION PARAMETERS

REST .95 NC, CF PTS. 222 NC. PARTIAL WAVES 12 POTENTIAL 1 MASS SHAPE 1 CHARGE SHAPE 2  
 PION ENERGY 75.00 MEV  
 MASS NUMBER 4.00 CHARGE = 2.00

## FINAL PARAMETERS FOR VARIABLES AND OTHER CONSTANTS

RE E0 = .2166E+01 + .4933E+01 F\*\*3  
 IM E0 = .5032E+02 + .8222E+01 F\*\*3  
 RE E1 = .6301E+01 + .1310E+02 F\*\*3  
 IM E1 = .7834E+02 + .1845E+02 F\*\*3  
 RE E2 = 0. F\*\*3  
 IM E2 = 0. F\*\*3  
 A = .1101E+01 + .1864E+01 F  
 T = .1P00E+01 F  
 AZ = .1463E+01 + .1624E+00 F  
 TZ = .1029E+01 F

## PI PLUS RESULTS

TOTAL	CROSSES SECTION	EXP = .1428E+23 MB	FIT = .1426E+23 MB	ERRCR = .7000E+01 MB	EXP-FIT = .6222E+00 MB
ANGLE		EXP MB/SR	FIT MB/SR	ERRCR MB/SR	EXP-FIT MB/SR
31.6		.9942E+01	.6209E+01	.2259E+00	.2622E+00
36.9		.5252E+01	.5234E+01	.1679E+00	.1644E+01
42.1		.4269E+01	.4109E+01	.1012E+00	.7999E+01
47.3		.3026E+01	.3167E+01	.1049E+00	.1621E+00
52.6		.9622E+02	.9783E+02	.2500E+01	.1834E+01
57.9		.6232E+02	.6409E+02	.1900E+01	.1792E+01
73.8		.4582E+02	.4975E+02	.1700E+01	.3992E+01
78.1		.5292E+02	.5326E+02	.1900E+01	.3633E+02
83.2		.7762E+02	.7207E+02	.2300E+01	.5533E+01
93.2		.1413E+01	.1422E+01	.4200E+01	.9283E+02
103.1		.2223E+01	.2368E+01	.5700E+01	.1647E+02
122.6		.4520E+01	.4307E+01	.1430E+00	.2011E+00
132.4		.5264E+01	.5085E+01	.1520E+00	.1787E+00
142.8		.6054E+01	.5704E+01	.1750E+00	.3499E+00
151.6		.6114E+01	.6163E+01	.2000E+00	.4660E+01

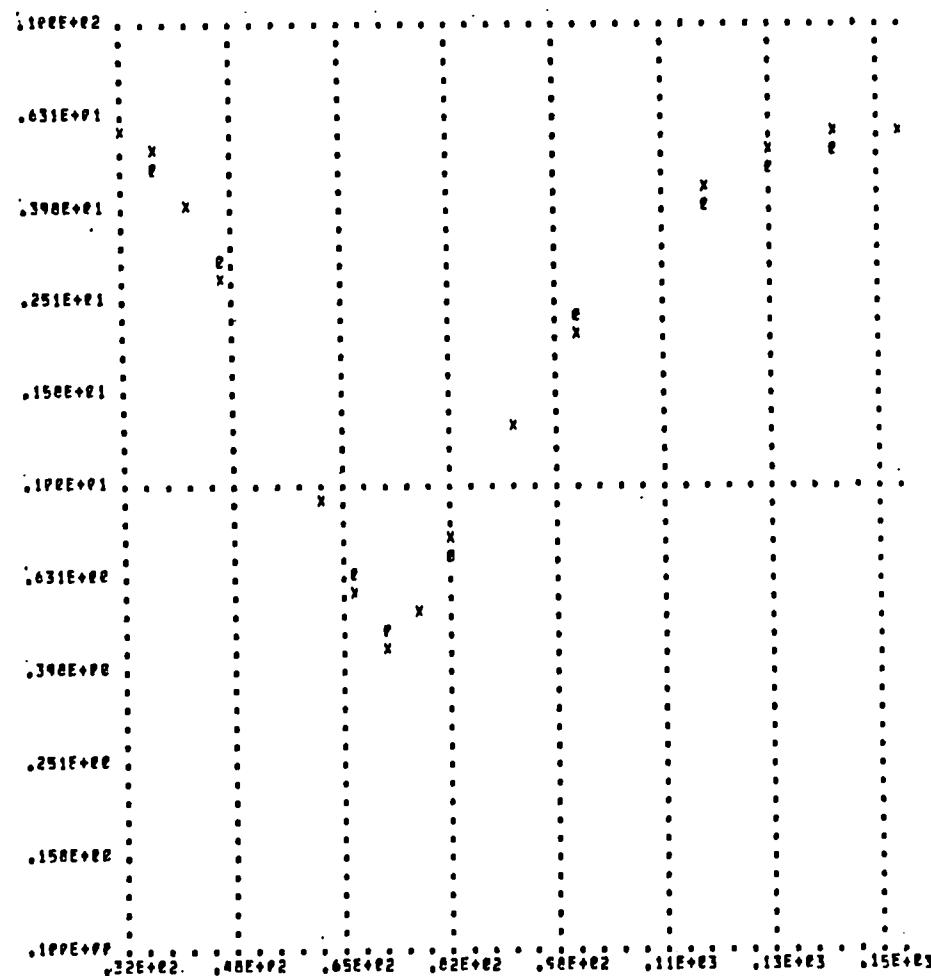
PI PI NLS RESULTS

TOTAL	CROSS SECTION	EXP = .1722E+23 MB	FIT = .1525E+23 MB	ERRCR = .6292E+21 MB	EXP-FIT = .1952E+22 MB
ANGLE		EXP MB/SR	FIT MB/SR	ERRCR MB/SR	EXP-FIT MB/SR
31.6		.9354E+21	.9450E+21	.2362E+22	.9566E-21
36.9		.7288E+21	.7316E+21	.2150E+22	.2360E+02
42.1		.5858E+21	.5515E+21	.1322E+22	.3432E+02
47.3		.3979E+21	.3950E+21	.1272E+22	.1477E-01
52.5		.1119E+21	.1097E+21	.2100E-01	.2273E+01
57.9		.6679E+22	.6651E+22	.1700E-01	.2147E+02
63.2		.4222E+22	.4756E+22	.1000E+01	.1235E-21
68.1		.4580E+22	.4880E+22	.1500F-01	.9174E-22
73.2		.7192E+22	.6756E+22	.1000E-01	.3422E+01
78.2		.1325E+21	.1411E+21	.3500E-01	.8561E-21
83.1		.2361E+21	.2422E+21	.5000E-01	.4727E+01
88.0		.4578E+21	.4041E+21	.1737E+22	.1365E+02
93.4		.5374E+21	.5250E+21	.1162F+22	.1295E+02
98.2		.5646E+21	.5885E+21	.1659E+22	.2426E+02
103.6		.6244E+21	.6366E+21	.2030E+22	.3139E+02

FINAL CHI-SQUARE = .7169E+22

## PI PLUS RESULTS

## SEPI=LOGARITHMIC PLCT SLEPCLTINE



PI WINS RESULTS

SEMI-LOGARITHMIC PLOT SUBROUTINE

